International Journal of Theoretical & Applied Sciences, 9(1): 35-42(2017)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

Potential Equations and Pressure Coefficient for Compressible Flow: Comparison between Compressible and Incompressible Flow in Aerodynamics

Menka Yadav[*] and Santosh Kumar Yadav[**]
[*]Research Scholar, J.J.T. University. Rajasthan, India
[**]Director (A&R), J.J.T. University, Rajasthan, India

(Corresponding author: (Corresponding author: Menka Yadav)
(Received 02 March, 2017 accepted 05 April, 2017)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: We derive the potential equation for slender bodies and seek to understand the flow field equations for subsonic, supersonic and transonic flow in framework of small perturbation. Large amount of heat and mass can be transferred in a efficient way between the surface and fluid when flow is released against the surface. When aircraft passes through several distinct regions, the flow develops a velocity and pressure profile. In stagnation-region large scale turbulent flow affects transfer coefficient. At the face of object total pressure is higher than behind the object. Profile slopes shows that compressible and incompressible flows are related via certain equations. Zero Mach number incompressible medium causes pressure disturbances to move uniformly in all directions. Flow of heat and mass transfer is strongly affected by the geometry of the device.

Keywords: Mach number, Pressure drag, Shock wave, Slender bodies, Velocity profile.

I. INTRODUCTION

Flow having significant changes in fluid density are known as compressible flow or flow with Mach number greater than 0.3 is treated as compressible. Compressible flow can be classified into subsonic, supersonic and hypersonic, based on the flow Mach number. This behaviour is mostly displayed by gases. In many field s like jet engines, high speed aircraft, rocket motors, high speed entry into a planetary atmosphere, gas pipelines etc., study of compressible flow is relevant. In testing of supersonic flows, supersonic wind tunnels are used.

In compressible flow, significant changes in velocity and pressure results in density variations throughout the flow field. Due to large temperature variations, density variations takes place. But in incompressible flow, material density remains constant. We make use of equations in flow field and tackle the problem by making some simplifications to the equation, depending on the type of flow to which it is to be applied. For certain flows, the equation can be reduced to an ordinary differential equation of a simple linear type. It can be reduced to nonlinear ordinary differential equation for some other type of flows. High pressure is created by the air flow over wing's bottom surface as compared to the top surface. Due to it, there is a net resultant force component, normal to the freestream. Flow direction, called lift, which acts on the wing. Velocity varies along the wing chord and in the direction normal to its surface. The region adjacent to the wall, where the velocity increases from zero to freestream value is known as the boundary layer. Viscous forces are predominant inside the boundary layer. The static pressure outside the boundary layer acting in the direction normal to the surface is trnasmitted to the boundary. Pressure coefficient becomes zero across boundary layer. If interlayer friction is neglected between the streamlines, outside the boundary layer, then it can be treated as invicid. Invicid flow is also called potential flow. Pressure can be invariant across the boundary layer and can be obtained in the field outside the boundary layer. Furthermore pressure in the freestream is impressed through the boundary layer. Fig. 1.

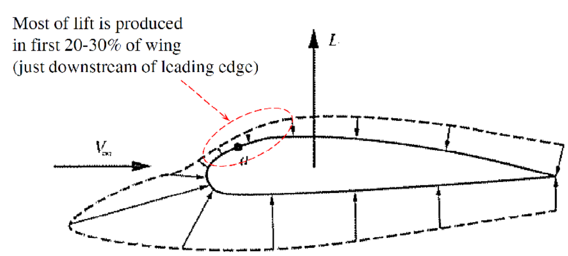

Fig. 1.

Airfoil generates lift due to imbalance of pressure distribution over the top and bottom surface of airfoil. Curved surface helps to reduce drag. In aerodynamics, static pressure is due to random motion of gas molecules and pressure we would feel if moving along with the flow, and stagnation (total) pressure exist if flow were slowed down isentropically to zero velocity. Combination of static and total pressure allows us to measure speed at a given point.

D =	D +	D
Total drag due to	Drag due to	Drag due to
separation		
Viscous effects	skin friction	
Called profile drag		

↕	↕
Less for laminar	More for laminar
More for turbulent	less for turbulent

II. Potential Equation For Subsonic, Supersonic And Transonic Flow For Slender Bodies

Continuity equation for a steady, invicid 3-D flow is:-

$$\nabla(\rho V)=0 \qquad (1)$$

The above equation can be written as

$$\rho\left(\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}\right)+V_x\frac{\partial \rho}{\partial x}+V_y\frac{d\rho}{\partial y}+V_z\frac{\partial \rho}{\partial z}=0 \qquad (2)$$

For isentropic process

$$\rho=\rho(P)$$

Velocity components in terms of ϕ can be written as:-

$$\left(1-\frac{\phi_x^2}{a^2}\right)\phi_{xx}+\left(1-\frac{\phi_y^2}{a^2}\right)\phi_{yy}+\left(1-\frac{\phi_z^2}{a^2}\right)\phi_{zz}-2\left(\frac{\phi_x\phi_y}{a^2}\phi_{xy}+\frac{\phi_y\phi_z}{a^2}\phi_{yz}+\frac{\phi_z\phi_x}{a^2}\phi_{zx}\right)=0 \qquad (3)$$

To obtain superposition of solution of potential equation for variable sound speed a, the above equation is:-

$$\left(\frac{a}{a_\infty}\right)=1-\frac{\gamma-1}{2}M_\infty^2\left(\frac{V_x^2+V_y^2+V_z^2}{V_\infty^2}-1\right) \qquad (4)$$

Flow around the aerofoil in terms of velocity components:-

$$\left.\begin{aligned}V_x&=V_\infty+u\\ V_y&=v\\ V_z&=w\end{aligned}\right] \qquad \ldots(5)$$

Small disturbances are introduced by slender or planar bodies. In this case perturbation velocities are less than main velocity components. Then:

$$V_x \approx V_\infty$$
$$V_y << V_\infty$$
$$V_z << w_\infty$$

For small angle of attack,

$$\begin{aligned}V_x&=V_\infty+u\\ V_y&=V\end{aligned} \qquad \ldots(6)$$

From equation (3) and (5), we have

$$(1-M^2)\phi_{xx}+\phi_{yy}+\phi_{zz}=0 \qquad \ldots(7)$$

Where M is Mach number and ϕ is perturbation potention and higher order terms are neglected. Using small perturbation theory and binomial theorem, relation between the local Mach number M and free stream Mach number M_∞ can be expressed as,

$$M^2=\left[1+2\frac{u}{V_\infty}\left(1+\frac{\gamma-1}{2}M_\infty^2\right)\right]M_\infty^2 \qquad \ldots(8)$$

Combining equation (7) & (8),

$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}+\phi_{zz}=\frac{2}{V_\infty}M_\infty^2\phi_x\phi_x\left(1+\frac{\gamma-1}{2}M_\infty^2\right) \ldots(9)$$

The above non-linear equation is valid for subsonic, transonic and supersonic flow. Equation (9) can be written as

$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}+\phi_{zz}=2\frac{M_\infty^2}{1-M_\infty^2}\frac{u}{V_\infty}\left(1+\frac{\gamma-1}{2}M_\infty^2\right)(1-M_\infty^2)\phi_{mm} \qquad \ldots(10)$$

If

$$\frac{M_\infty^2}{1-M_\infty^2}\frac{u}{V_\infty}<<1 \qquad \ldots(11)$$

Then linearization is possible and equation (10) becomes,

$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}+\phi_{zz}=0 \qquad \ldots(12)$$

Equation (11) is valid for subsonic and supersonic flow.

For $M_\infty \approx 1$, transonic flow equation (9) becomes

$$-\frac{(r+1)}{V_\infty}\phi_x\phi_{xx}+\phi_{yy}+\phi_{zz}=0 \qquad \ldots(13)$$

In terms of cylindrical polar coordinates, equation (9) can be written as

$$(1-M_\infty^2)\phi_{xx}+\phi_{rr}+\frac{1}{r}\phi_r+\frac{1}{r^2}\phi_{\theta\theta}=\frac{2}{V_\infty}M_\infty^2\phi_x\phi_{xx}\left(1+\frac{\gamma-1}{2}M_\infty^2\right) \quad \ldots(14)$$

Using condition of equation (11), equation (14) becomes

$$(1-M_\infty^2)\phi_{xx}+\phi_{rr}+\frac{1}{r}\phi_r+\frac{1}{r^2}\phi_{\theta\theta}=0 \quad \ldots(15)$$

Equation (15) is required equation for subsonic and supersonic flow in cylindrical coordinates.

For transonic flow, equation (14) becomes

$$\frac{\gamma+1}{V_\infty}\phi_x\phi_{xx}+\phi_{rr}+\frac{1}{r}\phi_r+\frac{1}{r^2}\phi_{qq}=0 \quad \ldots(16)$$

Equation (16) represents axially symmetric transonic flow.

III. Pressure Coefficient Of 3D Compressible Flow For Subsonic &Supersonic Flow

Friction causes flow separation and separation of body layer causes adverse pressure gradient which in turn arise pressure drag and overall drag is determined by body shape. The difference between local and freestream pressure is known as pressure coefficient.

Pressure distribution can be calculated by velocity distribution. Flow pattern, inside the circle is not influenced by the flow outside the circle. In irrotational flow, symmetry of pressure distribution shows that no drag is experienced by a steadily moving body. Pressure drag can be reduced for any shape by taking separation point as far as possible from the leading edge or forward stagnation point. Total pressure at the face of object is higher than total pressure behind the object.

Pressure coefficient C_p, for 3D flow is

$$C_p=\frac{2}{\gamma M_\infty^2}\left[1-\frac{\gamma-1}{2}M_\infty^2\left(\frac{2u}{V_\infty}+\frac{u^2+v^2+w^2}{V_\infty^2}\right)^{\frac{\gamma}{\gamma-1}}-1\right]$$

Where M_∞ = free stream Mach number

V_∞ = Free stream velocity

γ = Ratio of specific heat

u, v, w = *x, y&z* components of perturbation velocity Expanding binomially and neglecting higher order terms, we get,

$$C_p=-\left(2\frac{u}{V_i}+(1-M_\infty^2)\frac{u^2}{V_\infty^2}+\frac{v^2+w^2}{V_\infty^2}\right)$$

For planar bodies, C_P is

$$C_p=-2\frac{u}{v_\infty}$$

This fundamental equation is applicable for subsonic and supersonic.

IV. Relation Between Compressible And Incompressible Flow Using Perturbation Theory

Laplace equation for compressible flow is,

$$(1-M_\infty^2)\phi_{xx}+\phi_{zz}=0 \quad \ldots(1)$$

Laplace equation for incompressible flow is

$$(\phi_{xx})_{inc.}+(\phi_{zz})_{inc.}=0 \quad \ldots(2)$$

x and z coordinates are along and normal to the flow direction respectively.

Using boundary conditions, the above equations can be written as,

$$x_{inc}=x \quad \ldots(3a)$$

$$z_{inc}=k_{1z} \quad \ldots3(b)$$

$$\phi(x,z)=k_z\phi_{inc}(x_{inc.},z_{inc.})$$

From (1) & (3),

$$K_2\left[(1-M_\infty^2)\frac{\partial^2\phi_{inc.}}{\partial x_{inc.}^2}+K_1^2\frac{\partial^2\phi_{inc.}}{\partial z_{inc.}^2}\right]=0 \quad \ldots(4)$$

This is similar to incompressible potential equation:

If $$K_1=\sqrt{1-M_\infty^2} \quad \ldots(5)$$

Value of K_2 can be find out using boundary conditions.

In case of slender bodies, using perturbation theory, we have

$$\frac{w}{V_\infty+u}\approx\frac{w}{V_\infty}=\frac{dz}{dx} \quad \ldots(6)$$

But $\frac{u}{V_\infty}<<1$, the above equation can be written as

$$w=(\partial\phi/\partial z)=V_\infty\frac{\partial z}{\partial x} \quad \ldots7(a)$$

$$w_{inc.}=(\partial\phi_{inc.}/\partial z_{inc.})_{z_{inc.}=0}=V_\infty\frac{dz_{inc.}}{dx_{inc.}} \quad \ldots7(b)$$

From equation (3),

$$(\partial\phi/\partial z)_{z=0}=K_1K_2(\partial\phi_{inc.}/\partial z_{inc.})_{z_{inc.}}=0 \quad \ldots(8)$$

$$\frac{dz}{dx} = K_1 K_2 \left(\frac{dz_{inc.}}{dx_{inc.}} \right) \qquad (9)$$

$$\frac{dz}{dx} = K_2 \sqrt{1 - M_\infty^2} \left(\frac{dz_{inc.}}{dx_{inc.}} \right) \qquad \ldots(10)$$

From equation (10), it is clear that profile slope in compressible flow is $K_2\sqrt{1-M_\infty^2}$ times the slope in incompressible flow.

V. STUDY OF MOVING DISTURBANCE

Disturbance waves travels with local velocity of sound with respect to medium. Moving objects create pressure disturbance which propagates as shown in fig. below:-

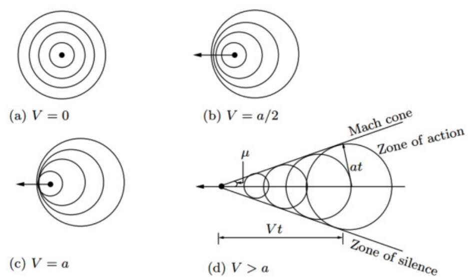

Fig. 2

The major difference between subsonic and supersonic flow is, different time taken by disturbance waves.
In subsonic flow, streamline detects the presence of obstacle earlier and flow smoothly around the obstacle. But in case of supersonic flow, presence of obstacle is detected only when hit by it. Moreover obstacle becomes source and streamlines are deviated at Mach cone. Surprisingly flow disturbance is sudden and flow behind the object changes quickly.

Flow for wedge is shown in fig. given below.

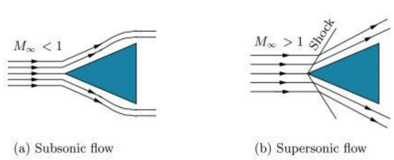

Fig. 3.

In fig.2(d), it is clear that conical zone is formed by object in supersonic flow, in which object is located at the nose and moving object is confined to the cone. No disturbance is felt by flow field, outside the zone. That is why it is termed as zone of silence. From fig. 2(d),

$$\sin\mu = \frac{1}{M}$$

where μ= Mach angle

It is Mach angle – Mach number relation

From fig. 1 we can conclude that

I. For incompressible medium, M=0 and pressure disturbance moves in all directions.
II. For subsonic speed, M<1, pressure disturbance is felt at every point of space with asymmetrical pressure pattern.
III. In case of supersonic speed M>1, pressure disturbance apex lies at its apex, included in a cone.

Disturbance waves can be considered identical to sound pulses when apex angle is very small. Small deviation in streamlines causes small increase in pressure in Mach cone. Finite deviation causes finite pressure across shock wave. Mach waves and Mach lines are linear and inclined at an angle μ in uniform supersonic flow and varies from point to point in non –uniform supersonic flow resulting Mach lines curved.

Normal, Shock Waves

Compression front supersonic flow across which the flow properties jump, and large gradients in temperature, pressure and velocity occur in thin region, where transport phenomena of momentum and energy are important, is known as Shock. In flow field, thickness of the shock is comparable to the mean free path of the gas molecules.

Quantitative Analysis of Normal Shock Wave

For quantitative analysis, let us consider an adiabatic, constant area flow through a non-equilibrium region, as shown in fig. given below. To define the flowproperties, let two sections 1 and 2, away from the non-equilibrium region.

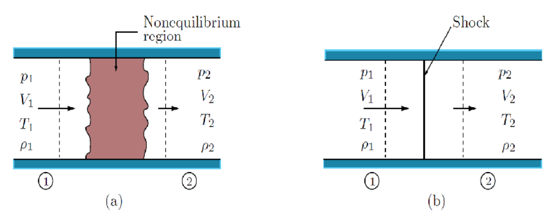

Fig. 4. Flow through a normal shock.

Equation of motion for the flow can be written as: From continuity equation,

$$\rho_1 V_1 = \rho_2 V_2 \ldots(1)$$

Momentum equation is:

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 \ldots(2)$$

Energy equation is: $h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 \ldots(3)$

The above mentioned equations are applicable to all gases and no restriction on the size, if the reference sections 1 and 2 are outside the non-equilibrium region. Relation between the flow parameters at these two sections can be obtained by solving these equations. Because there is a no restriction on non-equilibrium region, so it can be idealized as a thin region, as shown in fig.4(b). Heat traverses the shock wave, so it can neither added to nor taken away from the flow; hence flow process is adiabatic across the shock wave. Heat conduction in shock is provided by temperature and velocity gradients.

For a perfect gas, equation of state is:

$$P = \rho ST \ldots(4)$$

Enthalpy is $h = C_p T \ldots(5)$

Now, it is possible to obtain explicit solution in terms of Mach number as –

Dividing equation (2) and (1), we have

$$\frac{P_1}{\rho_1 V_1} - \frac{P_2}{\rho_2 V_2} = V_2 - V_1 \ldots(6)$$

As speed of sound $a = \sqrt{\gamma P/\rho}$ equation (6) becomes

$$\frac{a_1^2}{\gamma V_1} - \frac{a_2^2}{\gamma V_2} = V_2 - V_1 \ldots(7)$$

Using energy equations, we get

$$\frac{V_1^2}{2} + \frac{a_1^2}{\gamma-1} = \frac{V_2^2}{2} + \frac{a_2^2}{\gamma-1} = \frac{1}{2}\frac{\gamma+1}{\gamma-1}a^{*2}$$

a_1^2 and a_2^2 can be expressed as:

$$a_1^2 = \frac{\gamma+1}{2}a^{*2} - \frac{\gamma-1}{2}V_1^2$$

$$a_2^2 = \frac{\gamma+1}{2}a^{*2} - \frac{\gamma-1}{2}V_2^2$$

As the flow process across the shock wave is adiabatic, in a^* in the above relations for a_1^2 and a_2^2 has same constant value.

Substituting these relations in equation (7), we get

$$\frac{\gamma+1}{2}\frac{a^{*2}}{\gamma V_1} - \frac{\gamma-1}{2\gamma}V_1 - \frac{\gamma+1}{2}\frac{a^{*2}}{\gamma V_2} + \frac{\gamma-1}{2\gamma}V_2 = V_2 - V_1$$

$$\frac{\gamma+1}{2\gamma V_1 V_2}(V_2 - V_1)a^{*2} + \frac{\gamma-1}{2\gamma}(V_2 - V_1) = V_2 - V_1$$

Dividing above equation by $V_2 - V_1$, we get

$$\frac{\gamma+1}{2\gamma V_1 V_2}a^{*2} + \frac{\gamma-1}{2\gamma} = 1$$

or $a^{*2} = V_1 V_2 \qquad \ldots(8)$

which is called the Prandtl relation.

In terms of the speed ratio $M^* = \frac{V}{a^*}$, equation (8) can be written as:

$$M_2^* = \frac{1}{M_1^*} \ldots(9)$$

According to equation (9), velocity changes across a normal shock must be from supersonic to subsonic. Hence the Mach number behind a normal shock is always subsonic. Relation between the characteristic Mach number M^* and actual Mach number M is given by,

$$M^{*2} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2 + 2} \qquad \ldots(10)$$

This equation shows that the Mach number behind the shock is a function of only the Mach number M_1 ahead of the shock, for a perfect gas when $M_1 = 1$, $M_2 = 1$, then it becomes infinitely weak normal shock, identical to a Mach wave. Mach wave at $\mu = \sin^{-1}(1/M)$ is always less than $\pi/2$ in supersonic field. As M_1 is greater than 1, then the normal shock becomes stronger and M_2 becomes less than 1, and in this limit, as $M_1 \to \infty$, $M \to \sqrt{(\gamma-1)/2\gamma}$ with $\gamma = 1.4$ to 0.378.

$$\frac{V_1}{V_2} = \frac{V_1^2}{V_1 V_2} = \frac{V_1^2}{a^{*2}} = M_1^{*2} \ldots(11)$$

The ratio of velocities is,

and $$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2} \qquad \ldots(12)$$

Pressure relation can be obtained as

$$P_2 - P_1 = \rho_1 V_1^2 - \rho_2 V_2^2$$

From equation (1)

$$P_2 - P_1 = \rho_1 V_1 (V_1 - V_2)$$

$$= \rho_1 V_1^2 \left(1 - \frac{V_2}{V_1}\right)$$

Dividing above equation by P_1, we get

$$\frac{P_2 - P_1}{P_1} = \frac{\rho_1 V_1^2}{P_1}\left(1 - \frac{V_2}{V_1}\right)$$

as $a_1^2 = (\gamma P_1)/\rho_1$, we get

$$\frac{P_2 - P_1}{P_1} = \gamma M_1^2 \left(1 - \frac{V_2}{V_1}\right) \ldots(13)$$

Putting value of $\frac{V_2}{V_1}$ in above equation, we obtain

$$\frac{P_2 - P_1}{P_1} = \gamma M_1^2 \left[1 - \frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right] \ldots(14)$$

The above equation can be written as,

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M_2^2 - 1) \ldots(15)$$

ratio of $\frac{P_2 - P_1}{P_1} = \frac{\Delta P}{P_1}$ is called the shock strength.

The entropy change in terms of pressure and temperature ratios across the shock can be written as

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

as $$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \frac{a_2^2}{a_1^2} = 1 + \frac{2(\gamma+1)(\gamma M_1^2+1)(M_1^2-1)}{(\gamma+1)^2 M_1^2}$$

$$S_2 - S_1 = C_p \ln\left[1 + \frac{2(\gamma-1)\gamma(M_1^2+1)}{(\gamma+1)^2 M_1^2}(M_1^2-1)\right] - R\ln\left(1+\frac{2\gamma}{\gamma+1}(M_1^2+1)\right) \ldots(16)$$

The above equation shows that for a perfect gas with a given value of V, all variables are functions of M_1 only. This explains the importance of Mach number in the quantitative governance of compressible flows flow properties changes within a very short distance across the shock. This distance may be order of 10^{-5} cm. Hence the value of velocity and temperature gradients are very large within the shock structure. Hence entropy across the shock changes due to large gradients.

If the shock wave is not stationary, than neither the total enthalpy nor the total temperature are constant across the shock wave. Entropy varies with loss in pressure. In this case it becomes independent of velocity and there is nothing like stagnation entropy, so the entropy difference without any reference to the velocity level is,

$$S_2 - S_1 = R \ln \frac{P_{01}}{P_{02}} \ldots(17)$$

and

$$\frac{P_{01}}{P_{02}} = \left(1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right)^{-1/(\gamma-1)} \left\{\frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+1}\right\}^{\gamma/(\gamma-1)} \ldots(18)$$

It connects the stagnation pressure to the normal shock to flow Mach number ahead of the shock. In supersonic flow, when a pilot probe is placed, then a detached shock standing ahead of probe nose. Total pressure behind the detached shock is measured by the probe. We can find the flow Mach number ahead of the shock from equation (18), by knowing the stagnation pressure ahead of the shock, which is the pressure in the reservoir, for isentropic flow up to the shock.

Oblique Shock

Oblique shocks are essentially compression fronts across which the flow decelerates and the static pressure, temperature and density varies upto higher values. In deceleration, Mach number behind the shock is greater than unity, then the shock is termed weak oblique shock. If the downstream Mach number is greater than unity, then the shock is known as strong oblique shock. In practical flow, weak oblique shocks are formed. Strong oblique shocks are generated in the engine intakes of supersonic flight vehicles, where the engine has provision to control its backpressure. When the backpressure increases upto a definite value, the oblique shock at the engine inlet would becomes a strong shock and decelerate the supersonic flow passing through it to subsonic level.

Dependence of deflection angle θ on Mach number M_1 and shock angle β is,

$$\tan\theta = 2\cot\beta\left(\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma+\cos 2\beta)+2}\right)$$

This $\theta-\beta-M$ relation plays important role ion analysis of oblique shocks.

The expression on R.H.S. of above equation becomes zero at $\beta = \pi/2$ and $\beta = \sin^{-1}\left(\frac{1}{M_1}\right)$, which are the limiting value of β. In this range, deflection angle θ is positive and have a maximum value. For γ = 1.4, results obtained from above equation are plotted as:

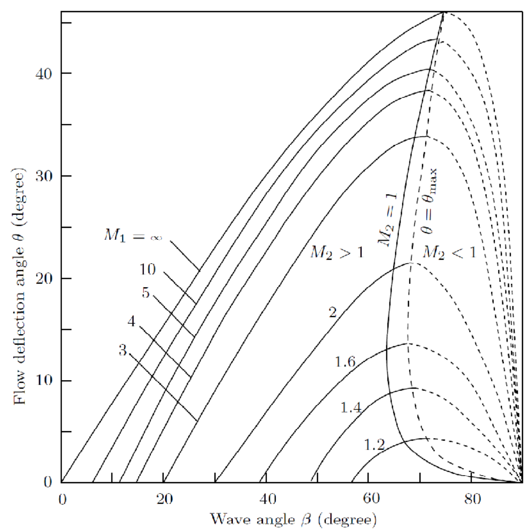

Fig. 5. Oblique Shock Solution.

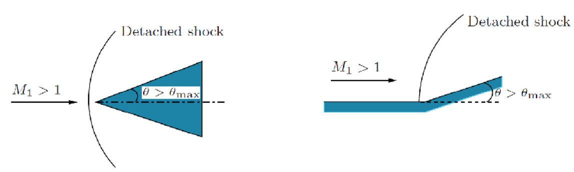

Fig. 6. Detached shocks.

Following conclusions can be drawn from the $\theta - \beta - M$ curves :

(a) There is a maximum value of θ, for any given supersonic Mach number M_1. If $\theta > \theta_{max}$, for given M_1, then no solution is possible for a straight oblique shock wave. In this case, the shocks shape will be curved and detached as shown in Fig.6(b).

(b) There are two possible solutions for $\theta > \theta_{max}$ for each value of θ and M, having two different wave angles. Large value of β is known as strong shock solution and for weak shock solution, the flow behind the oblique shock remains supersonic, except for small θ.

(c) Normal shock obtained for θ = 0 and β = π/2 and shock disappears and only Mach waves prevail in the flow field. When θ = 0, then two possible solutions are: (i) $\beta = \frac{\pi}{2}$ giving rise to a normal shock which does not cause any flow deflection but decelerates the flow to subsonic level (ii) $\beta = \sin^{-1}(1/M_1) = \mu$ corresponding a mach wave, inclined to upstream flow and would not cause any flow deflection, being the limiting case of the weakest isentropic wave for given M_1.

REFERENCES

[1]. Gunter Brenn, *Analytical Solution for Transport Process*, 2017.

[2]. A. Bhandarker et.al, *A Novel CFD method to estimate heat transfer coefficient for high speed flow* 2016.

[3]. Dhirendra Kumar Dixit, *Heat and Mass Transfer*, 2016.

[4]. F. Moukalled et.al, *the finite volume method in computational fluid dynamics*, 2016

[5]. Kurt C. Rolle, *Heat and Mass Transfer*, 2016.

[6]. Andrea de LietoVollaroet. al,: *CFD Analysis of Convective Heat Transfer Coefficient on External Surfaces of Buildings*, 2015.

[7]. IzzetSachin and AdemAcir, *Numerical and experimental investigations of Lift and Drag Performances of NACA 0015 Wind Turbine Airfoil*, 2015.

[8]. B. Sundenet. al, *Heat and Mass Transfer*, 2014.

[9]. E.L Cussler, *Diffusion Mass Transfer in Fluid systems*, 2013

[10]. Incropera, F.P., De Witt, D.P., Bergman, T.L., Lavine, A.S.: *Principles of Heat and Mass Transfer* 7thedn. Wiley, New York (2013)

[11]. S.S Bhat and R.N. Govardhan, "*Stall flutter of naca 0012 airfoil at low reynoldnumbers,* " *Journal of Fluids and Structures*, May,2013.

[12]. EthirajanRathakrishan*Theoretical Aerodynamics*, First Edition, 2013, John Wiley & Sons Singapor Pvt. Ltd.

[13]. D' Ambrosio D., "*A study on shock Wave – Boundary Layer Interactions in High Speed Flows* "4th European Symposium on Aerothermodynamics for Space Vehicles, October 2011.

[14]. D.Y. Shang, *Theory of Heat Transfer with Forced Convection Film Flows*, Springer Heidelberg, Dordrecht London, New York, 2011.

[15]. Dmitri Kuzmin, *A Guide to Numerical Methods for transport Equations*, 2010.

[16]. Rathakrishnan, E., *Applied Gas Dynamics*, John Wiley, NJ, 2010.

[17]. Recent Advances in *Heat Transfer Enhancements*: A Review Report, 2010.

[18]. J.F Wendt (ed.), *Computational Fluid Dynamics*, 3rd ed., Springer-verlag Berlin Heidelberg 2009.

[19]. S.K SOM, *Introduction to heat Transfer*, 2008.

[20]. J. Yao et.al, "*Numerical simulation of aerodynamics performance for two dimensional wind turbine airfoils*," 2011.

[21]. Keith J. Moss, *Heat and Mass Transfer*, 2007.

[22]. C.P. Kothandaraman, *Fundamentals of Heat and Mass Transfer*, 2006.

[23]. NiranjanSahoo et.al,:*Simultaneous measurement of aerodynamic and heat transfer data for large angle blunt cones in hypersonic shock tunnel*, 2006

[24]. Carslow, H.S., Jaeger, J.C.: *Conduction of Heat in Solids*, 2nd edn. Oxford Science Publications, Oxford (2005)

[25]. Di Clemente M., Marini M., Schettino a, "*Shock Wave Boundary Layer Interactions in SCIROCCO Plasma Wind Tunnel, East –West High Speed Flows Conference*, Pechino, October 2005

[26]. Di Clemente M., MARINI M., Schettino A, "*Shock Wave Boundary Layer Interaction in EXPERT Flight Conditions and Scirocco PWT*" , 13th AIAA/ CIRA International Space planes conference Capua , may 2005.

[27]. Neil Zuckerman, *Impingement Heat Transfer: Correlations and Numerical Modelling*, May 2005.

[28]. Bird, R.B., Stewart, W.E. and Lightfoot, E.N., *Transport Phenomena* 2nd edition, Wiley, 2004.

[29]. De Filippis F., Caristia S., Del Vecchio A., Purpura C., "*The Scirocco PWT Facility Calibration Activities*", *3rd international Symposium Atmospheric Re- entry Vechile and Systems, Arcachon*, France, March, 2003.

[30]. Francisco Javier Trujillo et.al, *CFD Modelling of the heat and mass transfer process during the evaporation of water from a circular cylinder*, 2003.

[31]. T. Cebeci, *Convective Heat Transfer* 2ndedn. (Springer, Heidelberg, 2002)

[32]. A.D. Kraus, A. Aziz, and J.R Welty, *Extended Surface Heat Transfer*, John Wiley & sons, New York, NY, USA, 2001.

[33]. I. Pop, D.B. Ingham, *Convective heat transfer– Mathematical and Computational Modelling of Viscous Fluids and Porous Media* (Elsevier, Amesterdam, 2001).

[34]. Marini M., "*Analysis of hypersonic compression ramp laminar flows under sharp leading edge conditions, Aerospace, Science and Technology*, no. **5**, 2001.

[35]. Dujardin A., Guelhan A., Longo J.M.A., Mack A., "*Numerical /Experimental Investigation of a Wedge-Compression Corner with a Gap- Flow under cold-Hypersonic Conditions*", Proceedings of the 4th European Symposium on Aerothermodynamics for Space Vehicle, October 2001.

[36]. Davis J.P., Sturtevant, B., "*Separation Length in High -Enthalpy Shock/ Boundary Layer Interaction*", "Physics of Fluids, Vol. **12**, No. 10, 2000.

[37]. G.K Batchelor, *an Introduction to Fluid Dynamics*, Cambridge University Press, 2000.

[38]. J.W. Rose, *Condensation Heat Transfer* Int. J. Heat Mass Transfer, 1999.
[39]. Shames I.H. *Mechanics of Fluids*, 3rd Ed. McGraw Hill, New York, 1992.
[40]. Harris, C. D., *NASA Supercritical Aerofoil– A Matrix of Family Related Airfoils*, NASA Technical paper 2969, 1990.
[41]. Magnus, A.E., And Epton, M.A., "*PAN AIR-A computer program for predicting subsonic or supersonic linear potential flows about arbitrary configurations using a higher order panel method*", Vol.-1, Theory Document, NASA,1980.